Quantization leading to a natural flattening of the axion potential

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Abstract

Starting from the general cosine form for the axion effective potential, we quantize the axion and show that the result is described by a naturally flat potential, if interactions with other particles are not considered. This feature therefore restores the would-be Goldstone-boson nature of the axion, and we calculate the corresponding vacuum energy density, which does not need to be too large by orders of magnitude compared to Dark Energy.

Introduction. It is known that a quantized scalar theory must be described by a convex one-particle-irreducible (1PI) potential [1], as a consequence of the so-called spinodal instability, where the restoration force on field fluctuations vanishes. In the situation of a bare potential presenting a concave region, as a Mexican-hat potential, the corresponding 1PI potential can be obtained by a procedure similar to the Maxwell construction in Thermodynamics [2]. This property is valid for a self-interacting scalar field only, and therefore does not apply to a situation where the scalar also interacts with other degrees of freedom, such as the Higgs boson in the Standard Model.

An example of the self-interacting case is provided by the Sine-Gordon model, in 1+1 dimensions, where Wilsonian renormalization studies have been done, dealing with the corresponding spinodal instability [3]. In these studies, the authors consider the coarse grained potential, which interpolates the bare potential and the 1PI potential, when the cut off of the theory decreases. They find that, between two minima of the cosine bare potential, the coarse grained potential flattens when the cut off decreases, and becomes flat in the IR limit. We note here that this flatness holds in a wider area than the one where the bare potential is concave, and the flat 1PI potential joins the minima of the bare potential. As a consequence, starting from a cosine bare potential, the only way for the 1PI potential to be convex is to be flat. Other studies involving a bare cosine potential, and the corresponding flattening due to spinodal instability, have been made in the context of quitessence [4]. The authors consider a description based on spinodal decomposition, using a potential function of two variables: the mean scalar field and its fluctuation condensate, in the framework of a Hartree approximation.

We consider here a cosine potential, in 3+1 dimensions, which is characteristic of axions, and we show how a flat potential indeed arises. As a consequence, starting with a cosine bare potential, the axion self-interactions gradually flatten the potential, and we are left with a vacuum energy only, which we calculate. A novel feature of the present study is that we quantize the axion degrees of freedom as compared to regarding the axion as a classical field in conventional cosine-based expressions for the potential. Using this fully quantized treatment we derive an expression for the self-interacting axion vacuum energy and we see that in some

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heavy axion models, this vacuum energy is not necessarily orders of magnitude larger than observed and estimated values for Dark Energy. Coupling the axion to other particles of the Standard Model, though, can prevent the flattening of the potential, and we discuss a specific example, where the axion is coupled to fermions. We derive conditions to avoid the spinodal instability, such that no flattening of the axion potential occurs.

This article is structured as follows. We first review the quantization of the axion model, in order to define our notations, and we show that the corresponding 1PI potential must be convex. We then derive an exact self consistent equation for this 1PI potential, in the form of a differential Schwinger-Dyson equation, which is non-perturbative. A flat 1PI potential is indeed a solution of this equation, and we calculate the corresponding vacuum energy density. Finally, We discuss the coupling of the axion to other particles.

The axion and its quantization. In the original paper [5], the axion is introduced to build a CP conserving theory, from a model with massive fermions coupled to a non-Abelian gauge field, where a CP violating term arises at one-loop due to non-perturbative QCD instanton effects, [10]. This original model was refined following experimental and cosmological evidence and [7] proposes a more general Lagrangian with three potentially non-zero coupling constants from which the original work [5], or invisible axion models such as [11], can be constructed. In this paper we use the term axion in this general sense, defined as initially arising as a phase - a Goldstone mode - of a complex scalar field, Φ which acquires a vacuum expectation value $\langle\Phi\rangle = fe^{i\theta}$ due to spontaneous breaking of a global $U(1)$ symmetry at a scale f. Explicit symmetry breaking occurs at some scale μ, with $\mu < f$ at which point the axion can acquire mass. Motivated by the need to solve the CP problem, we require that the axion model possess a non-zero anomalous gluon coupling term $\theta G\widetilde{G}$. References [7], [8], and [9] quote the commonly used general expression for the axion potential as $K(1-\cos\theta)$, where K has mass dimension 4. In arriving at this expression the initial step is integrating over the QCD degrees of freedom. The total partition function is

$$Z = \int \mathcal{D}[\phi,\Phi]\exp\left(-S[\phi,\Phi]\right) = \int \mathcal{D}[\theta]\exp\left(-S_\theta\right), \tag{1}$$

where ϕ represents the QCD degrees of freedom and $\Phi = \rho e^{i\theta}$. S_θ defines then the effective action for the axion, which we will quantize. The calculation of S_θ is necessarily approximate, and an analysis of the form of the axion potential in light of new understandings of the QCD vacuum, using supersymmetric gauge and brane theories, is presented in [9]. The authors note that the $K(1-\cos\theta)$ form for the potential of the QCD axion needs not be limited to one cosine dependence, but may have higher harmonics and in general is a smooth periodic function of θ with period 2π. They conclude that with axion models using light quarks $m_q \ll \Lambda_{QCD}$ one arrives at a cut-off independent potential of the form $K(1-\cos\theta)$.

Motivated by these results and the general form of the Lagrangian presented in [7], we use as our general form for the axion action as follows.

$$S_\theta = \int d^4x\left\{\frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + \sum_{n=1}^{\infty} a_n(1-(\cos\theta)^n)\right\}, \tag{2}$$

where a_n are coefficients to be determined (units quartic in mass). The action (2) can be understood as a consequence of the upgrading of the phase of $\langle\Phi\rangle$ to a dynamical field, that we

wish to quantized. An example of quantization of the axion is done in [6], where the authors consider quantum fluctuations of the axion, leading to a Bose-Einstein condensate which could play the role of cold Dark Matter.

In terms of the canonically normalized field $\phi = f\theta$, this theory is not renormalizable, and we will need to keep the cut off Λ throughout the study, which will be considered a parameter of the theory, as well as f. Also, by definition of the cut off, we will always assume that $f \leq \Lambda$. (For completeness, we also assume that $\Lambda_{QCD} \leq \Lambda$.)

Quantization of the model (2) is based on the partition function.

$$Z[j] = e^{-W[j]} = \int \mathcal{D}[\theta] \exp\left(-S_\theta - \int d^4x \; j\theta\right), \tag{3}$$

from which the classical field is defined as

$$\theta_{cl} \equiv \frac{\delta W}{\delta j} = \langle \theta \rangle \,, \tag{4}$$

where

$$< \cdots >= \frac{1}{Z} \int \mathcal{D}[\theta](\cdots) \exp\left(-S_\theta - \int d^4x \; j\theta\right). \tag{5}$$

A second functional derivative with respect to the source gives

$$-\frac{\delta^2 W}{\delta j \delta j} = \langle \theta\theta \rangle - \theta_{cl}\theta_{cl}, \tag{6}$$

which is the variance of the set of variables $\theta(x)$, therefore positive, such that

$$\frac{\delta^2 W}{\delta j \delta j} \leq 0, \tag{7}$$

which will be used to show the convexity of the 1PI potential. The proper graph generator functional $\Gamma[\theta_{cl}]$ is defined as the Legendre transform of the connected graphs generator functional $W[j]$:

$$\Gamma[\theta_{cl}] = W[j] - \int d^4x \; j \; \theta_{cl}, \tag{8}$$

where the source j is understood as a functional of θ_{cl}, and we have then

$$\frac{\delta \Gamma}{\delta \theta_{cl}} = -j \qquad \text{and} \qquad \frac{\delta^2 \Gamma}{\delta \theta_{cl} \delta \theta_{cl}} = -\left(\frac{\delta^2 W}{\delta j \delta j}\right)^{-1}. \tag{9}$$

The convexity of the 1PI potential U follows directly from the previous properties [1]. U is the non derivative part of Γ: if θ_0 is a constant classical field, $\Gamma[\theta_0] = \int d^4x U(\theta_0)$. The corresponding source, denoted j_0, is also constant, such that the relation between second functional derivative in eqs.(9) gives then, after integration over space time,

$$\frac{d^2 U}{d\theta_0^2} = -\int d^4x \left(\frac{\delta^2 W}{\delta j \delta j}\right)^{-1}_{j_0}, \tag{10}$$

such that, taking the inequality (7) into account, we have, for any value of θ_0,

$$\frac{d^2U}{d\theta_0^2} \geq 0, \tag{11}$$

and the potential is necessarily convex. As a consequence, if the bare potential is concave, at least in a certain range of field values, quantum corrections necessarily contain tree-level contributions, in order to compensate the concave features of the bare potential and make the 1PI potential convex.
If the axion is coupled to other degrees of freedom, then $\delta^2 W$ has to be understood as a matrix, with rows and columns corresponding to the different degrees of freedom. The second derivative of the 1PI potential is then an element of the inverse matrix $(\delta W)^{-1}$, which is not necessarily positive. In this latter situation, concave features of the bare potential can be stable, and quantum corrections are perturbative only.
Note that, if one considers a periodic axion field, which is restricted to the interval $[0; 2\pi]$ in the path integral (3), the latter should be defined with

$$\int \mathcal{D}[\theta] \equiv \Pi_x \int_0^{2\pi} d\theta(x), \tag{12}$$

but the derivation of the property (7) would not change. Indeed, this inequality was not obtained by any explicit path integration, but corresponds to a generic property of $Z[j]$. For the same reason, the existence of non-trivial vacua and topological features in the classical theory does not alter the property (7): all the corresponding information is contained in the path integral. Only a saddle point approximation would have to take into account these topological features, but no approximation or expansion is made here.
Finally, by construction, the effective potential must be differentiable, since it is obtained by integrating over the original field θ. The only way for the effective potential to be convex, differentiable and periodic is to be flat: we will see now that this solution is indeed satisfied.

Self consistent equation for the 1PI potential. Our approach consists in studying the evolution of the proper graph generator functional Γ with the amplitude of the decay constant f. This functional approach is inspired from the original version given in [12], and will lead us to an exact partial differential equation for the 1PI potential U. In a first stage, this procedure can be seen as a mathematical tool, to obtain a non-perturbative self-consistent equation for U. Nevertheless, the physical meaning of the approach is the following. Suppose we start from $f >> m$, the interaction is then negligible compared to the kinetic term, and the theory is almost free: quantum fluctuations are "frozen". As f decreases, quantum fluctuations start to dress the system, which will eventually describes the proper quantum theory when f reaches its physical value. This argument will be used to define the boundary condition necessary to solve the self-consistent differential equation that we now derive.
From now on, we represent a derivative with respect to f with a dot. We have then

$$\dot{\Gamma} = \dot{W} + \int d^4x \frac{\delta W}{\delta j} \partial_f j - \int d^4x \; \partial_f j \; \theta_{cl} = \dot{W}, \tag{13}$$

where the latter derivative can be obtained from the bar action (2), such that

$$\dot{W} = f \int d^4x \, \langle \partial^\mu \theta \partial_\mu \theta \rangle \; . \tag{14}$$

In what follows, we omit the subscript cl for the classical field. From eq.(6), we obtain then

$$\dot{\Gamma} = f \int d^4x d^4y \; \delta^{(4)}(x-y) \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y_\mu} \left\{ \theta_x \theta_y - \frac{\delta^2 W}{\delta j_x \delta j_y} \right\}, \tag{15}$$

and, using eq.(9), we finally find that the exact evolution equation of Γ with f is

$$\dot{\Gamma} = f \int d^4x \; \partial^\mu \theta \partial_\mu \theta + f \int d^4x d^4y \; \delta^{(4)}(x-y) \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y_\mu} \left(\frac{\delta^2 \Gamma}{\delta \theta_x \delta \theta_y} \right)^{-1}, \tag{16}$$

where the trace contains all the quantum corrections, and is regularized by the cut off Λ. We stress that this equation is exact: it is a self-consistent equation for Γ, which is independent of any perturbative expansion.
Next step is to consider an ansatz for Γ, and we take here

$$\Gamma = \int d^4x \left(\frac{f^2}{2} \partial^\mu \theta \partial_\mu \theta + U(\theta) \right), \tag{17}$$

which allows an f-dependent 1PI potential $U(\theta)$. It is then sufficient to consider a constant axion configuration: the evolution of the effective potential is then given by $\dot{\Gamma} = \mathcal{V} \dot{U}(\theta)$, where $\mathcal{V}$ is the volume of space time. We have thus

$$\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y_\mu} \left(\frac{\delta^2 \Gamma}{\delta \theta_x \delta \theta_y} \right)^{-1} = \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{-p^\mu q_\mu}{f^2 p^2 + U''} \delta^{(4)}(p+q) e^{-ipx-iqy}, \tag{18}$$

such that, taking into account $\mathcal{V} = \delta^{(4)}(p=0)$,

$$\int d^4x d^4y \; \delta^{(4)}(x-y) \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y_\mu} \left(\frac{\delta^2 \Gamma}{\delta \theta_x \delta \theta_y} \right)^{-1} = \mathcal{V} \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{f^2 p^2 + U''(\theta)}, \tag{19}$$

where a prime denotes a derivative with respect to the axion. The integration over p leads then to the following non-perturbative and self-consistent equation for the 1PI potential

$$\dot{U}(\theta) = \frac{1}{16\pi^2 f} \left[\frac{\Lambda^4}{2} - \frac{\Lambda^2}{f^2} U''(\theta) + \frac{1}{f^4} [U''(\theta)]^2 \ln \left(1 + \frac{f^2 \Lambda^2}{U''(\theta)} \right) \right], \tag{20}$$

where Λ is the UV cut off used to regularize the theory. There are several solutions to the non-linear partial differential equation (20), but, as expected from the spinodal instability discussed above, a flat potential U_{vac} is indeed a solution, and satisfies

$$\dot{U}_{vac} = \frac{\Lambda^4}{32\pi^2 f} \tag{21}$$

We consider some key points at this stage:

- The flattening of the potential cannot be found with a perturbative expansion, but only with a non-perturbative approach: quantum fluctuations are so dominant in such a system, that they wipe off the classical non-convex features of the bare potential;

- The cosines in the bare potential do not appear anywhere in the evolution equation (20), because the latter is a self-consistent equation for the dressed potential. The cosines of the bare potential will play a role as a boundary condition used to solve this differential equation;

- The axion, originally a Goldstone boson, and acquiring mass after integration of QCD degrees of freedom, eventually indeed becomes massless, after quantization of the model (2);

- The flat potential U_{vac} is not related to some vacuum expectation value of the axion field, and therefore does not need to satisfy any specific transformation under parity;

- The equation (21) can be written as a time-energy uncertainty relation, $f\tau = 1$, where

$$\tau = 32\pi^2 \frac{\dot{U}_{vac}}{\Lambda^4}. \tag{22}$$

  Hence, although we are dealing with a field theory at equilibrium, one might interpret τ as the time that quantum fluctuations need to flatten the potential. This interpretation gives a physical meaning to the quantity $\dot{U}_{vac}/\Lambda^4$.

Boundary condition. The integration of eq.(21) necessitates a boundary condition $f = \Lambda \Rightarrow U = U_\Lambda$, such that

$$U_{vac} = U_\Lambda + \frac{\Lambda^4}{32\pi^2} \ln\left(\frac{f}{\Lambda}\right). \tag{23}$$

In order to determine U_Λ, we use the following argument. When $f = \Lambda$, the bare potential expressed in terms of the canonically normalized field $\phi = \Lambda\theta$ is

$$V(\phi) = \sum_{n=1}^{\infty} a_n (1 - (\cos\theta)^n) = \frac{1}{2}\frac{m^4}{\Lambda^2}\phi^2 + \mathcal{O}\left(\frac{\phi}{\Lambda}\right)^4, \tag{24}$$

where

$$m^4 \equiv \sum_{n=1}^{\infty} n a_n. \tag{25}$$

Assuming $m << \Lambda$, the interactions are negligible compared to the kinetic and mass energies: the corresponding dressed flat potential U_Λ is then given by its one-loop expression, which is exact if the bare potential (24) is considered quadratic. The flattening of the potential is a tree level effect, and therefore of zeroth order in $\hbar$: without further corrections, this would lead to a flat potential interpolating the minima of the classical potential, at zero energy. The one-loop correction, of first order in $\hbar$, is then identified with U_Λ,

$$\begin{aligned} U_\Lambda &= \frac{1}{2}\mathrm{Tr}\left\{\ln\left[(-\partial^2 + V'')\delta^{(4)}(x-y)\right]\right\} \\ &= \frac{\Lambda^4}{64\pi^2}\left[\left(1 - \frac{m^8}{\Lambda^8}\right)\ln\left(1 + \frac{\Lambda^4}{m^4}\right) - \frac{1}{2} + \frac{m^4}{\Lambda^4}\right] \\ &\simeq \frac{\Lambda^4}{16\pi^2}\ln\left(\frac{\Lambda}{m}\right), \end{aligned} \tag{26}$$

such that the flat potential (23) is finally

$$U_{vac} \simeq \frac{\Lambda^4}{32\pi^2} \ln\left(\frac{f\Lambda}{m^2}\right). \tag{27}$$

To be consistent with the interpretation in terms of vacuum energy, we need $U_{vac} \geq 0$, such that the following constraint should be satisfied

$$f\Lambda \geq m^2, \tag{28}$$

which is a property common to every axion model. Note that the naive estimate of the vacuum energy density ρ_{vac}, based on the bare cosine potential, is

$$\rho_{vac} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + \frac{m^4}{f^2}} \simeq \frac{\Lambda^4}{16\pi^2}. \tag{29}$$

Although the ratio $f\Lambda/m^2$ is in principle large, the result (27) is actually of the same order of magnitude than the estimate (29), because of the logarithm of this ratio. However, for heavy axions [13], the ratio $f\Lambda/m^2$ can actually be of order 1 given $m = m(f)$, see for example [14] where m is quadratic in f. In such a situation, the vacuum energy (27) can be much smaller than the naive estimate (29), such that the contribution of the axion vacuum energy is not necessarily orders of magnitude larger than observed and estimated values for Dark Energy. We note attempts to link the axion with Dark Energy [15] and that our result (27) can be a model-independent starting point for such links.

Interactions with other particles and discussion. One could argue that, because of the flatness of the potential, the initial motivation for the axion is lost, since no restoring force traps the axion in some vacuum expectation value, which compensates CP violation. In order to prevent this, we now consider a more realistic scenario where the axion is coupled to leptons with mass μ, via the term [16]

$$gf\theta\overline{\psi}\gamma^5\psi \tag{30}$$

where g is a dimensionless coupling, and we will discuss the conditions under which the spinodal instability can be avoided. Integrating out leptons, we obtain the effective potential (before quantizing of the axion):

$$V_{eff}(\theta) = \sum_{n=1}^{\infty} a_n(1 - (\cos\theta)^n) - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\ln\left(\frac{p^2 + \mu^2 + g^2f^2\theta^2}{p^2 + \mu^2}\right), \tag{31}$$

where the integral is regulated by the fermion cut off M. The spinodal instability, present with the cosine potential only, is avoided if the second derivative of the effective potential (31) is positive, such that the axion does not need to become massless. The original cosine potential is concave at $\theta = \pi$ for example, where we evaluate the second derivative of the effective potential (31):

$$\begin{aligned} \frac{d^2V_{eff}}{d\theta^2}(\pi) &= -m^4 - \frac{g^2f^2M^2}{16\pi^2}\,\frac{M^2 + \mu^2 + 3g^2f^2\pi^2}{M^2 + \mu^2 + g^2f^2\pi^2} \\ &\quad + \frac{g^2f^2}{16\pi^2}(\mu^2 + 3g^2f^2\pi^2)\ln\left(1 + \frac{M^2}{\mu^2 + g^2f^2\pi^2}\right) \end{aligned} \tag{32}$$

and we are left with the following situations (we neglect the mass μ):

- if $M >> gf$, the second derivative (32) is

$$V''_{eff} \simeq -m^4 - \frac{g^2 f^2 M^2}{16\pi^2} < 0, \tag{33}$$

  and the spinodal instability is not avoided. This is expected for a large fermion cut off, since fermions then have a negative contribution to the axion effective potential, and do not improve convexity;

- if $M \simeq gf\pi$, then

$$V''_{eff} \simeq -m^4 + c^4 g^4 f^4, \tag{34}$$

  where $c = [(3\ln 2 - 2)/16]^{1/4} \simeq 1/4$, and the spinodal instability can thus be avoided if gf is larger than the threshold $\simeq 4m$, which is realistic, given the phenomenological constraints [16].

Note that, if the axion interacts with a scalar degree of freedom Φ, via the coupling $lf^2\theta^2\Phi^2/4$, where l is a dimensionless coupling, the correction (33) becomes

$$V''_{eff} \simeq -m^4 + \frac{lf^2 M^2}{32\pi^2}, \tag{35}$$

and can be positive, if l is not too small, in which case no concave feature needs to be compensated by quantum fluctuations. This discussion can be extended to other interactions with the Standard Model, for example by coupling the axion to the photon via the interaction

$$\frac{h}{4}\theta F^{\mu\nu} F^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \tag{36}$$

where h is a dimensionless coupling.

The essential results from this study can be summarised as follows.

- When one takes into account quantum fluctuations of the axion, spinodal instabilities result in a flattening of the effective potential and a massless axion. We derive a model-independent form for the resulting vacuum energy which need not be orders of magnitude higher than estimates for dark energy.

- In this quantized axion model, when axion couplings to other particles are considered, we find that spinodal instabilities can be avoided, allowing the axion to acquire mass as is the case with conventional axion models.

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References

[1] Y. Fujimoto, L. O'Raifeartaigh and G. Parravicini, Nucl. Phys. B **212** (1983) 268; R. W. Haymaker and J. Perez-Mercader, Phys. Rev. D **27** (1983) 1948.

[2] C. Wetterich, Nucl. Phys. B **352** (1991) 529; J. Alexandre, V. Branchina and J. Polonyi, Phys. Lett. B **445** (1999) 351 [arXiv:cond-mat/9803007].

[3] I. Nandori, J. Polonyi and K. Sailer, Phys. Rev. D **63** (2001) 045022 [arXiv:hep-th/9910167]; I. Nandori, K. Sailer, U. D. Jentschura and G. Soff, J. Phys. G **28** (2002) 607 [arXiv:hep-th/0202113]; I. Nandori, U. D. Jentschura, K. Sailer and G. Soff, Phys. Rev. D **69** (2004) 025004 [arXiv:hep-th/0310114]; S. Nagy, J. Polonyi and K. Sailer, J. Phys. A **39** (2006) 8105; S. Nagy, I. Nandori, J. Polonyi and K. Sailer, Phys. Lett. B **647** (2007) 152 [arXiv:hep-th/0611061]; Phys. Rev. D **77** (2008) 025026 [arXiv:hep-th/0611216]; Phys. Rev. Lett. **102** (2009) 241603 [arXiv:0904.3689 [hep-th]]; I. Nandori, S. Nagy, K. Sailer and A. Trombettoni, Phys. Rev. D **80** (2009) 025008 [arXiv:0903.5524 [hep-th]]; V. Pangon, S. Nagy, J. Polonyi and K. Sailer, arXiv:0907.0496 [hep-th]; V. Pangon, arXiv:1008.0281 [hep-th]; I. Nandori, S. Nagy, K. Sailer and A. Trombettoni, arXiv:1007.5182 [hep-th]; I. Nandori, arXiv:1008.2934 [hep-th].

[4] D. Cormier and R. Holman, Phys. Rev. Lett. **84** (2000) 5936 [arXiv:hep-ph/0001168]; S. Tsujikawa and T. Torii, Phys. Rev. D **62** (2000) 043505 [arXiv:hep-ph/9912499].

[5] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16** (1977) 1791. R. Peccei and H. Quinn, Phys. Rev. Lett. **38** (1977) 1440-1443; S. Weinberg Phys. Rev. Lett. **40** (1978) 223-226;

[6] P. Sikivie and Q. Yang, Phys. Rev. Lett. **103** (2009) 111301, [arXiv:0901.1106v4]

[7] J. E. Kim, Rev. Mod. Phys. **82**, (2010)557601 J. Kim, Phys. Rev. Lett. **43** (1979) 103-107; J. Kim, G. Carosi, Rev. Mod. Phys. **82**, 557601 (2010); J. Kim Phys. Rep. **150**, (1987) 1177

[8] P. Svrcek and E. Witten, JHEP **051** (2006) 0606; [arXiv:hep-th/0605206].

[9] G. Gabadadze, M. Shifman, Int. J. Mod. Phys. **A17** (2002) 3689-3728; [arXiv:hep-ph/0206123]

[10] G. 't Hooft, Phys. Rev. **D 14**, 34323450 (1976); G. 't Hooft, Phys. Rev. Lett. **37**, (1976) 8-10; C. Callan, R. Dashen and D. Gross, Phys. Rev. D **19**, (1979) 18261855.

[11] J. E. Kim, Phys. Rev. Lett. **43** (1979) 103; M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B166** (1980) 493;

[12] J. Alexandre and J. Polonyi, Annals Phys. **288** (2001) 37 [arXiv:hep-th/0010128].

[13] S. H. H. Tye, Phys. Rev. Lett. **47** (1981) 1035; V. A. Rubakov, JETP Lett. **65** (1997) 621 [arXiv:hep-ph/9703409];

[14] P. Sikivie, Lect. Notes Phys. **741**(2008) 19-50; [arXiv:astro-ph/0610440];

[15] J. Kim, H. Niles Phys. Lett. **B553** (2003) 1-6; [arXiv:hep-ph/0210402] J. Kim, H. Niles JCAP 0905:010 (2009), [arXiv:hep-th/0902.3610]

[16] K. Zioutas *et al.* [CAST Collaboration], Phys. Rev. Lett. **94** (2005) 121301 [arXiv:hep-ex/0411033]; E. Masso and J. Redondo, Phys. Rev. Lett. **97** (2006) 151802 [arXiv:hep-ph/0606163]; G. G. Raffelt, Lect. Notes Phys. **741** (2008) 51 [arXiv:hep-ph/0611350]; Y. Nomura and J. Thaler, Phys. Rev. D **79** (2009) 075008 [arXiv:0810.5397 [hep-ph]]; M. Y. Khlopov, A. S. Sakharov and D. D. Sokoloff, Nucl. Phys. Proc. Suppl. **72** (1999) 105.